MC

1. **A**

Solution: After simplifying the equation by cancelling out numbers, n^2 = 5 x 5 x 9. Thus n = 5 x 3 = 15.

(Pascal 2018, #5).

2. **A**

Solution: The question implies that the value of S - T will be the same regardless of what 10 integers we use. Thus we use the integers 1 through 10: In this case, S = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55 and T = 10(1) = 10 or S − T = 45

(Pascal 2006, #19).

3. **E**

Solution: Since w is a positive integer, then $w \neq 0$, so $w^3 = 25w$ implies $w^2 = 25$ (we can divide by w since it is non-zero). Since $w^2 = 25$, $w = 5$ (since w must be a positive integer). Thus $w^5 = 5^5 = 3125$

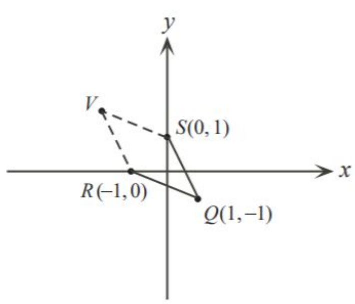
(Cayley 2006, #17).

4. **A**

Solution:

We plot the first three vertices on a graph.

We see that one possible location for the fourth vertex, V , is in the second quadrant:



If VSQR is a parallelogram, then SV is parallel and equal to QR.

To get from Q to R, we go left 2 units and up 1 unit.

Therefore, to get from S to V , we also go left 2 units and up 1 unit.

Since the coordinates of S are (0, 1), then the coordinates of V are (0 − 2, 1 + 1) = (−2, 2).

This is choice (A).

There are two other possible locations for the fourth vertex, which we can find in a similar way.

These are U(0, −2) and W(2, 0).

Using these points, we can see that SQUR and SW QR are parallelograms. But (0, −2) and (2, 0) are not among the possible answers.

Therefore, of the given choices, the only one that completes a parallelogram is (−2, 2).

(Cayley 2011, #15).

5. **B**

Solution:
Connie gives 24 bars that account for 45% of the total weight to Brennan. Thus, each of these 24 bars accounts for an average of 45
24% = 15
8 % = 1.875% of the total weight.
Connie gives 13 bars that account for 26% of the total weight to Maya. Thus, each of these 13 bars accounts for an average of 26
13% = 2% of the total weight.
Since each of the bars that she gives to Blair is heavier than each of the bars given to Brennan (which were the 24 lightest bars) and is lighter than each of the bars given to Maya (which were the 13 heaviest bars), then the average weight of the bars given to Blair must be larger than 1.875% and smaller than 2%

Note that the bars given to Blair account for 100% − 45% − 26% = 29% of the total weight.
If there were 14 bars accounting for 29% of the total weight, the average weight would be
29
14% ≈ 2.07%, which is too large. Thus, there must be more than 14 bars accounting for 29% of the total weight.
If there were 15 bars accounting for 29% of the total weight, the average weight would be
29
15% ≈ 1.93%, which is in the correct range.
If there were 16 bars accounting for 29% of the total weight, the average weight would be
29
16% ≈ 1.81%, which is too small. The same would be true if there were 17 or 18 bars.
Therefore, Blair must have received 15 bars.

(Cayley 2010, #23)

Word Problems

1. **6a^3 - 13a^2 + 4**

Solution:
(a − 2)(6a^2 − a − 2)
= 6a^3 - a^2 - 2a - 12a^2 + 2a + 4
= 6a^3 - 13a^2 + 4

(CSMC 2013, #2)

2. $PM = 2\sqrt{13}$ **and** $RN = \sqrt{73}$

Solution:
Since PQ = 6 and N is the midpoint of P Q, then PN = NQ = 3.
Since QR = 8 and M is the midpoint of QR, then QM = MR = 4.

Since ΔPQM is right-angled at Q and PM > 0, we can apply the Pythagorean Theorem to obtain:

$$PM = \sqrt{PQ^2 + QM^2} = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$$

Since ΔNQR is right-angled at Q and RN > 0, we can apply the Pythagorean Theorem to obtain:

$$RN = \sqrt{NQ^2 + QR^2} = \sqrt{3^2 + 8^2} = \sqrt{73}$$

Therefore, the two medians have lengths $PM = 2\sqrt{13}$ and $RN = \sqrt{73}$

(CSMC 2012, #3)

3.

Solution:

Let x-1, x, and x+1 represent the three consecutive integers. The sum of their squares is equal to: $(x-1)^2 + x^2 + (x+1)^2 = x^2 - 2x + 1 + x^2 + x^2 + 2x + 1 = 3x^2 + 2$. Now we need to prove why there cannot be an integer z such that $z^2 = 3x^2 + 2$.

Notice how if we divide the right hand side of the equation by 3, the result would be a remainder of 2. This asks the question: What would happen if we divide z^2 by 3? Is it possible for a perfect square to have a remainder of 2 when divided by 3?

Now we have three cases to consider: $z = 3k$, $z = 3k + 1$, $z = 3k - 1$ for integer *k*.

If $z = 3k$, then $z^2 = 9k^2$, which when divided by 3, results in a remainder of 0
If $z = 3k + 1$, then $z^2 = 9k^2 + 6k + 1$, which when divided by 3, results in a remainder of 1
If $z = 3k - 1$, then $z^2 = 9k^2 - 6k + 1$, which when divided by 3, results in a remainder of 1

Thus, it is impossible for a perfect square to have a remainder of 2 when divided by 3, only a remainder of 1 or 0 is possible. Therefore, it is impossible for the sum of the squares of three consecutive integers to be a perfect square.