MC

1. **D**

Solution: Evaluating, the inside is equal to 5^2 - 4^2 = 9, and the square root of 9 is 3, so the answer is D. .

(Cayley 2016, #3).

2. **C**

Solution: p^2 is an odd * odd, so the number will be odd. Adding an even number will result in only odd numbers.

3. **B**

Solution: Let a be the unknown integer. The equation will be in the form: (a+1)^2 - (a)^2 = 1401. We solve that a = 700. So, the addition of 701^2 + 700 ^2 = 981401 is the solution.

4. **D**

Solution: If we imagine it as a slice of pizza, everytime we add another term, we are cutting the distance between the last slice and “12 o'clock” position by half. Eventually, we will be approaching 2.

5. **903/1024 (not one of the choices)**

Solution:

Let q = 1 - answer, i.e., q is the probability that at least one team wins/loses all 7 games it plays.

Note that the probability that a given team wins/loses all 7 games it plays is $2 \times (1/2)^7 = 1/64$. Since there are 8 teams, we may be tempted to say that q is equal to $8 \times (1/64) = 1/8$. However, this double-counts cases where there are at least two teams such that each of them wins/loses all 7 games it plays, so we need to subtract the probability p that this happens from 1/8 to obtain q.

Now, we find p. Note that we can’t have two teams that win all the games they play (they can’t win against each other at the same time), and similarly, we can’t have two teams that lose all the games they play. Therefore, it is impossible to have more than two teams such that each of them wins/loses all games it plays. Therefore, p is simply the probability that there are exactly two teams such that each of them wins/loses all games it plays. This can only happen if one of the two teams (e.g. team A) wins all 7 games it plays and the other (e.g. team B) loses all 7 games it plays. For this to happen, A wins against B, A wins against C, A wins against D, A wins against E, …, B loses against C, B loses against D, B loses against E, …, which has probability $1/2^{7+7-1} = 1/2^{13}$. There are 8 possibilities to choose a team that wins all its games, and afterwards there are 7 possibilities to choose a team that loses all its games. Therefore,

$$p = 8 \times 7 \times (1/2^{13}) = 7/2^{10} = 7/1024.$$

Now, we have $q = 1/8 - p = 1/8 - 7/1024 = 121/1024$ and our answer is

$$1 - q = 1 - 121/1024 = 903/1024.$$

(Fermat 2017, #24)

Word Problems

1. Since zero is a root of $x^3 - 4x = 0$, the product must be zero.

(CSMC 2012, A2)

2. We list the twelve integers from smallest to largest:
1277, 1727, 1772, 2177, 2717, 2771, 7127, 7172, 7217, 7271, 7712, 7721
The sum of the 7th and 8th integers in the list is 7127 + 7172 = 14299.

(CIMC 2017, A2)

3. If the sum of the original sequence is $\sum_{i=1}^{n} a_i$ then the sum of the new sequence can be expressed as

$$\sum_{i=1}^{n} a_i + (2i-1) = n^2 + \sum_{i=1}^{n} a_i.$$

Therefore, $836 = n^2 + 715 \to n = 11.$ Now the middle term of the original sequence is simply the average of all the terms, or $\frac{715}{11} = 65,$ and the first and last terms average to this middle term, so the desired sum is simply three times the middle term, or $\boxed{195}$.

(AIME I 2012, #2)

Challenge:
See https://www.youtube.com/watch?v=6v0QqJ87gdM.