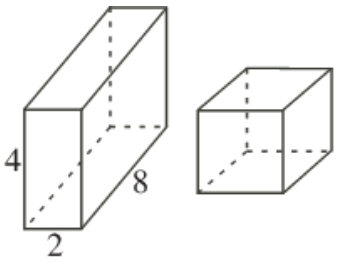
Would you like to receive feedback from our execs? (marking your solutions, giving corrections, etc.)
Yes/No: _____

Multiple Choice

Highlight the correct answer for each question.

1. The rectangular cuboid and the cube in the diagram have equal volumes. Each edge of the cube has length:



(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

2. In a right-angled triangle, the sum of the squares of the three side lengths is 1800. The length of its hypotenuse is

(A) √1800 (B) ½ √1800 (C) 90 (D) 30 (E) 45

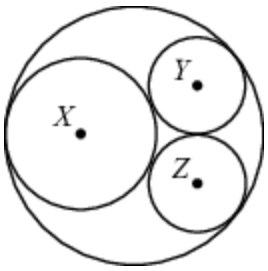
3. The symbols ♥ and ∇ represent different positive integers less than 20. If ♥ × ♥ × ♥ = ∇, what is the value of ∇ × ∇?

(A) 12 (B) 16 (C) 36 (D) 64 (E) 81

4. A bag contains eight yellow marbles, seven red marbles, and five black marbles. Without looking in the bag, Igor removes *N* marbles all at once. If he is to be sure that, no matter which choice of *N* marbles he removes, there are at least four marbles of one colour and at least three marbles of another colour left in the bag, what is the maximum possible value of *N*?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

5. In the diagram, the circle with center *X* is tangent to the largest circle and passes through the center of the largest circle. The circles with centers *Y* and *Z* are each tangent to the other three circles, as shown. The circle with center *X* has radius 1. The circles with centers *Y* and *Z* each have radius *r*. The value of *r* is closest to



(A) 0.93 (B) 0.91 (C) 0.95 (D) 0.87 (E) 0.89

Word Problems

Either type your solutions or insert images of handwritten solutions. Be sure to show your work for full credit!

1. If $12x = 4y + 2$, determine the value of the expression $6y - 18x + 7$.

2. f is a function that takes a real number x with $0 \le x \le 1$ with these properties:

1. $f(1 - x) = 1 - f(x)$
2. $f(x / 3) = f(x) / 2$
3. $f(a) \le f(b)$ for all reals $0 \le a \le b \le 1$

Find the value of $f(6/7)$.

3. Oh no! Zed has woken up only to discover he has forgotten the natural numbers! Fortunately, Zed has not forgotten how to use Google so he searches up a way to construct them. He finds that a natural number is:

- zero or
- the *successor* of another natural number n, which is written as **succ**(n)

For example, one is *defined* to be **succ**(zero), two is *defined* to be **succ**(**succ**(zero)), etc.

Zed wants to do useful things with these new numbers and so finds this definition of addition:

For two natural numbers a and b, **add**(a, b) is *defined* as follows

- if b = zero, then **add**(a, b) = a
- otherwise if b = **succ**(c) for some natural number c and **add**(a, b) = **succ**(**add**(a, c))

Zed notices is this oddly *asymmetrical*. He also notices **add** is partly defined using **add** itself. (This is called a recursive definition.) Being skeptical, he adds one and two:

1. Help carry out this calculation for Zed. He has already started a bit:
   add(one, two) = **add**(one, **succ**(one))
   = **succ**(**add**(one, one))
   = ...

2. Zed wonders what **add**(zero, n) is for some natural number n so he decides to calculate **add**(zero, two). Help Zed do this as well.

3. Zed conjectures that **add**(zero, n) = n for *all* natural numbers n. Prove Zed's conjecture.
   Note: This is not true by definition but **add**(n, zero) = n is true by definition.
   Hint (written in white):

Survey

Your responses will not affect your likelihood of being counted for attendance. This is simply to let our execs know how we can improve. :)

1. Approximately how much time did you spend on this problem set?

(A) Less than 15 mins
(B) 15 mins to 30 mins
(C) 30 mins to 1 hour

(D) 1 to 2 hours
(E) Over 2 hours

2. How difficult did you find this problem set?

(A) Too easy
(B) Fairly easy
(C) Neutral
(D) Fairly difficult
(E) Too difficult

Thank you for your feedback!