MC

1. **B**

Solution: Disregarding the 6's, we are simply just counting up by five. Adding 5 to 14 gets us to 19.

2. **E**

Solution: Since $3 \times n = 6 \times 2$, then $3n = 12$ or $n = 12/3 = 4$.

(Cayley 2018, #1)

3. **D**

Solution: Since the face of a foonie has area 5 cm^3 and its thickness is 0.5 cm, then the volume of one foonie is $5 \times 0.5 = 2.5$ cm^3. If a stack of foonies has a volume of 50 cm^3 and each foonie has a volume of 2.5 cm^3, then there are $50 \div 2.5 = 20$ foonies in the stack.

(Cayley 2011, #9)

4. **D**

Solution: The averages of groups of three numbers are equal if the sums of the numbers in each group are equal, because in each case the average equals the sum of the three numbers divided by 3. Therefore, the averages of three groups of three numbers are equal if the sum of each of the three groups are equal. The original nine numbers have a sum of

$$1 + 5 + 6 + 7 + 13 + 14 + 17 + 22 + 26 = 111$$

and so if these are divided into three groups of equal sum, the sum of each group is $111/3 = 27$. Consider the middle three numbers. Since two of the numbers are 13 and 17, then the third number must be $37 - 13 - 17 = 7$. We note that the remaining six numbers can be split into the groups 5, 6, 26 and 1, 4, 22, each of which also has a sum of 37. Therefore, the number that is placed in the shaded circle is 7.

(Pascal 2017, #21)

5. **C**

Solution: Initially, all 100 cards have the red side up. After Barsby's first pass only the 50 odd-numbered cards have the red side up, since he has just turned all the even-numbered cards from red to yellow.

During Barsby's second pass he turns over all cards whose number is divisible by 3. On this pass Barsby will turn any odd-numbered card divisible by 3 from red to yellow. Between 1 and 100, there are 17 odd numbers that are divisible by 3, namely 3, 9, 15, 21, … , 93, and 99. Also on this pass, Barsby will turn any even-numbered card divisible by 3 from yellow to red. Between 1 and 100, there are 16 even numbers that are divisible by 3, namely 6, 12, 18, 24, … , 90, and 96.

When Barsby is finished, the cards that have the red side up are the 50 odd-numbered cards from the first pass, minus the 17 odd-numbered cards divisible by 3 from the second pass, plus the 16 even-numbered cards divisible by 3, also from the second pass.

Thus, $50 - 17 + 16 = 49$ cards have the red side up.

(Fermat 1988, #20)

Word Problems

1. **24**

Solution 1: Any positive integer that is a multiple of 8 is also a multiple of each of 2 and 4, because 8 itself is a multiple of 2 and 4. Therefore, we are looking for the smallest positive integer that is a multiple of 6 and 8.
$1 \times 8 = 8$ is not a multiple of 6.
$2 \times 8 = 16$ is not a multiple of 6.
$3 \times 8 = 24$ is a multiple of 6, and so is the smallest positive integer that is a multiple of each of 2, 4, 6, and 8.

Solution 2: Any positive integer that is a multiple of 8 is also a multiple of each of 2 and 4, because 8 itself is a multiple of 2 and 4. Therefore, we are looking for the smallest positive integer that is a multiple of 6 and 8. This integer is the least common multiple (LCM) of 6 and 8. We note that $6 = 2 \times 3$ and $8 = 2^3$.
The least common multiple of 6 and 8 can be obtained by first looking at the prime factors of these integers. The only prime factors of 6 and 8 are 2 and 3. The factor 3 occurs a maximum of 1 time (in 6) and the factor 2 occurs a maximum of 3 times (in 8).
Thus, the least common multiple of 6 and 8 is $3 \times 2^3 = 24$.

(CIMC 2013, #2)

2. **The 6th bucket**

Solution 1: Let b represent the number of buckets after the first.
Since the first bucket contains 17 green discs and each bucket after the first contains 1 more green disc than the previous bucket, then there are $17 + b$ green discs inside the bucket b after the first.
Since the first bucket contains 7 red discs and each bucket after the first contains 3 more red discs than the previous bucket, then there are $7 + 3b$ red discs inside the bucket b after the first.
The number of green discs in a bucket is equal to the number of red discs inside the same bucket when $17 + b = 7 + 3b$ or when $2b = 10$, and so when $b = 5$.
Thus, there are an equal number of red discs and green discs in the fifth bucket after the first bucket, which is the 6th bucket.

Solution 2: Using the fact that the first bucket contains 17 green discs and 7 red discs, and each bucket after the first contains 1 more green disc and 3 more red discs than the previous bucket, then we may summarize the number of green and red discs inside each bucket.

Bucket Number	Number of green discs	Number of red discs
1	17	7
2	18	10

3	19	13
4	20	16
5	21	19
6	22	22

Therefore, the 6th bucket contains an equal number of red discs and green discs.

Solution 3: In the first bucket, there are 17 green discs and 7 red discs. Each bucket after the first contains 1 more green disc and 3 more red discs than the previous bucket.
Since there are 2 more red discs than green being put in, then the difference between the numbers of green and red discs will decrease by 2 for each bucket after the first.
Since the original difference is 17 – 7 = 10, then it takes 10 ÷ 2 = 5 more buckets to arrive at a bucket where the numbers of green and red sics will be equal.
Therefore, the 6th bucket contains an equal number of red discs and green discs.

(Galois 2016, #1b)

3. 14

We start by considering the ones (units) column of the given sum. From the units column, we see that the units digits of 3*C* must equal 6. The only digit for which this is possible is *C* = 2. Thus, the sum becomes

$$\begin{array}{r} B\,2 \\ A\,B\,2 \\ +\;\;A\,B\,2 \\ \hline 8\;7\;6 \end{array}$$

We note that there is no “carry” from the ones column to the tens column.
Next we consider the tens column. From the tens column, we see that the units digit of 3*B* must equal 7. The only digit for which this is possible is *B* = 9. Thus, the sum becomes

$$\begin{array}{r} 9\,2 \\ A\,9\,2 \\ +\;\;A\,9\,2 \\ \hline 8\;7\;6 \end{array}$$

The “carry” from the tens column to the hundreds column is 2.
Next, we consider the hundreds column. We see that the units digit of 2*A* + 2 (the two digits plus the carry) must equal 8. Thus, the units digit of 2*A* must equal 6. This means that *A* = 3 or *A* = 8. The value of A = *8* is too large as this makes *A*92 = 892 which is larger than the sum. Therefore, *A* = 3. Therefore, *A* + *B* + *C* = 3 + 9 + 2 = 14.

(CSMC 2015, #2)